

Scientific Studies of Magic Squares

Peter Loly

Department of Physics, The University of Manitoba,
Winnipeg, Manitoba Canada R3T 2N2 (loly@cc.umanitoba.ca)

July 27, 2003

Abstract

While magic squares are recreational in grade school, they may be treated somewhat more seriously in linear algebra courses. The smallest and earliest is the ancient Chinese 3×3 Lo-shu magic square of the numbers 1..9, which has a magic line constant of 15 for rows, column, and diagonals. This square motif appears in the same time frame as yin-yang schema (I Ching, Feng Shui) in early Chinese cosmology and philosophy. Over the past few years I have made several novel contributions, often through work with undergraduate summer students, finding pandiagonal non-magic squares, turning the compounding method into a computer code to produce the world's largest magic square, obtaining theorems for the moment of inertia and electric quadrupole tensors, and Monte Carlo estimates of populations of these squares. I will show very fruitful interdisciplinary interactions between mathematics, computing (digital design), physics (mechanics and electricity) and philosophy (Carl Jung's depth psychology).

1 Introduction

Small magic squares are often encountered in early grades as an arithmetic game on square arrays (patterns or motifs). Classical magic squares of the whole numbers, $1..n^2$, have the same line sum (magic constant) for each row, column, and main diagonals:

$$C_n = n(n^2 + 1)/2 \tag{1}$$

This line sum invariance depends only on the order, n , of the magic square. The ancient 3×3 Chinese Lo-shu square of the first nine consecutive integers is the smallest magic square and apart from rotations and reflections there are no others this size or smaller:

4	9	2
3	5	7
8	1	6

(2)

where the magic sum is 15. For $n = 3$, $C_3 = 15$, as expected. The best scientific statement that can be made about its age is 2500 ± 1500 years! While the middle figure may be most appropriate, the hardest evidence gives just the lesser (Ollerenshaw and Brée, 1998), while legends (Swetz, 2002) claim the older.

We briefly review the recreational aspect of magic squares before drawing attention to the simpler semi-magic squares, and to pandiagonal non-magic squares. Then we examine the scientific aspects of all these squares through applications in classical physics and matrix analysis. In fact it is through the coupled oscillator problem that the mathematics of matrices can be introduced for their eigenproperties. Magic squares can then play a valuable role in modern courses in linear algebra.

2 Recreational Mathematics

At the recreational level magic squares are fun for all ages, as I found when introducing them to visitors during the summer of 2000 whilst volunteering at the "Arithmetricks" travelling exhibit at the Museum of Man and Nature in Winnipeg, Manitoba. Various types of magic squares have become a recreational pastime of amateurs, often very gifted individuals e.g. Albrecht Dürer, and Ben Franklin. There are several journals which publish results in this area of recreational mathematics. However magic squares present difficult challenges for mathematicians and over the past few hundred years many famous mathematicians have contributed to our knowledge of them, including Euler.

3 Semi-Magic and Pandiagonal Magic Squares

For a number of purposes it is important to recognize that there are two specially important variations on the theme of magic squares.

Firstly, **semi-magic squares**, which do not have the diagonals summing to the row-column line sum, some of which may be obtained simply by moving an edge row and/or column to the opposite side, e.g.

$$\begin{array}{|c|c|c|} \hline 9 & 2 & 4 \\ \hline 5 & 7 & 3 \\ \hline 1 & 6 & 8 \\ \hline \end{array} \tag{3}$$

The removal of the row and column constraints means more squares due to the smaller number of constraints, in this case there are eight more. Secondly, **pandiagonal non-magic squares**, which have the same magic line sum for all the split lines parallel to the main diagonals. We can illustrate pandiagonals by taking a non-magic serial square and tiling a copy to its right (or left, or top or bottom):

$$\begin{array}{|c|c|c|c|c|c|} \hline 1 & 2 & 3 & 1 & 2 & 3 \\ \hline 4 & 5 & 6 & 4 & 5 & 6 \\ \hline 7 & 8 & 9 & 7 & 8 & 9 \\ \hline \end{array} \tag{4}$$

The pandiagonals are (2, 6, 7), (3, 4, 8), (1, 6, 7), (2, 4, 9), together with the main diagonals. Observe that for a given order the number of row and column constants is the same as the number of pandiagonal constraints.

A pandiagonal magic square is the combination of this pandiagonal property with the requirements of a magic square and 48 of these are found amongst the 4th order squares, with none existing for singly even orders (6, 10, ...):

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

(5)

4 Counting Magic Squares

There are 880 distinct 4×4 magic squares of the first 16 integers, and the 275,305,224 distinct 5×5 of the first 25 integers, the latter were first counted by computer in 1973 (Schroeppel). Already by order six they have become uncountable, and as a result only statistical estimates are then possible. Pinn and Wieczerkowski (1998) performed a Monte Carlo simulated annealing computation for an estimate of $(0.17745 \pm 0.00016) \times 10^{20}$ for the 6×6 , and a less accurate estimate of the 7×7 .

Walter Trump (2003) has developed a more efficient hybrid backtracking Monte Carlo method and has improved the accuracy of these estimates. He also has good estimates of the number of magic squares up to 10×10 , a remarkable achievement. Our group of summer undergraduate students has recently taken Trump's 6×6 GB32 code, which uses 13 random cells, and converted it to C++. Matt Rempel finds that the C++ code runs about 10% faster than GB32 on the same PC in producing a sample of some 700,000 squares, and has begun to remove random cells, finding longer run times, but more accurate results, and a larger sample of magic squares. Dan Schindel has taken the ideas in the 6×6 code and constructed pure backtracking codes without random cells to count the known numbers of magic squares for the 4×4 and 5×5 cases. While we have previously been able to analyse the complete set of 4×4 's, we can now begin to analyse the 5×5 's. This gives us the ability to study a variety of interesting questions, e.g. their eigenproperties. Dan Schindel has also amended the 4×4 code to count the number of pandiagonal non-magic 4×4 squares, finding some three million.

5 Compound Squares

An undergraduate project with Wayne Chan took an old Chinese idea for compounding a 3×3 magic square with itself to construct a 9×9 magic square, or a 3×3 with a 4×4 to make a pair of 12×12 magic squares, and devised a computer program (Chan and Loly, 2002) to extend this to very large squares. One of the squares is used as a frame and the other is incremented on placement

in the appropriate position in the frame. As a result we were able to set a new world record sized magic square at $12,544 \times 12,544$.

6 Where is the Science?

Why did a theoretical physicist get involved with magic squares? This was not a linear process. Having been blissfully unaware of them for my first five decades, an encounter with the Myers-Briggs Type Indicator (MBTI) scheme of personalities resonated with my background in mathematical structures, partly from research in solid state physics. Coordinate rotation matrices in classical and relativistic mechanics, combined with periodic boundary conditions for finite crystals soon lead to links with magic squares, while the psychological nature of the MBTI eventually made connections with early work of Carl Jung, which finally connected with Jung's interest in Chinese patterns, especially the dichotomous yin-yang schemes of Feng Shui (a.k.a. The Golden Flower) and the I Ching {von Franz, 1974}.

The science begins whenever we go beyond the square patterns, e.g. treating magic squares as matrices, or as arrays of point masses or electric charges. There are more examples, but our own work concentrates on the remarkable results associated with matrix issues in the context of linear algebra, i.e. solving sets of simultaneous equations, as well as some topics in classical physics.

7 A Mechanical Application - Moment of Inertia

Another success growing out of teaching undergraduate classical mechanics for many years was the discovery of a new invariance, or universal property, for magic squares through calculations of their "moments of inertia" (essentially the inertia of the square through an axis perpendicular to its centre), which eventually turned out to depend only on the order of the square, i.e. the number of rows or columns (Loly, 2004). The numbers in the magic square are replaced by corresponding multiples of a unit mass placed on a square unit lattice. In fact this was truly a "Eureka" moment for quite spontaneously I had the idea which fused long activity in teaching moment of inertia in introductory courses with a more recent activity in magic squares.

The moment of inertia, I_n , of a magic square of order n about an axis perpendicular to its centre is obtained by summing mr^2 for each cell, where m is the number centred in a cell and r is the distance of the centre of that cell from the centre of the square measured in units of the nearest neighbour distance. For the Lo-shu square the corner cells then have their centres at a distance of $\sqrt{2}$ from the axis. We can now calculate the sum for the 3×3 :

$$I_3 = [1 + 3 + 7 + 9] (1)^2 + [2 + 4 + 6 + 8] (\sqrt{2})^2 = 60 \quad (6)$$

The moments of inertia about the horizontal and vertical axes through the centre are each 30, reminding us of the perpendicular axis theorem, which says that their sum gives the moment of inertia about the axis through the centre and perpendicular to the plane.

When I used a data file for the complete set of the 4×4 's (by courtesy of Harvey Heinz) it was a surprise to find that they all gave $I_4 = 340$. I was then motivated to attempt a derivation, which was easy since the calculations only depended on the semi-magic property so that the parallel axis theorem and the perpendicular axis theorem could be used. In retrospect this could have been set as an examination problem in a sophomore course on classical mechanics!

$$I_n = \frac{1}{12}n^2(n^4 - 1) \quad (7)$$

This remarkably simple formula recovers the results for $n = 3, 4$ above and is valid for arbitrary order. The derivation of the formula only depends on the row and column properties, and not on the diagonals of magic squares, so that it actually applies to the larger class of **semi-magic** squares which lack one or both diagonal magic sums of magic squares.

Adam Rogers has recently extended these ideas to the calculation of the full inertia tensor of magic cubes (Rogers and Loly, 2003).

8 Electric Quadrupoles

A new magic square topic has just emerged from my renewed involvement with our honours electromagnetism course. The idea is to treat the numerical value of each element as an electric charge. It is soon clear for small squares that the dipole moment vanishes, so we then proceed to study the quadrupole moment. As a first thought I neutralized magic squares so that their elements ran from $-(n^2 - 1)/2$ to $+(n^2 - 1)/2$, but later Adam Rogers analyzed the multipole expansion for a normal magic square, finding that it takes care of many of the details. The full story involves calculating the quadrupole tensor, something beyond the scope of the present article. The short story is that the quadrupole tensor vanishes, so one then proceeds to the octupole!

9 Pandiagonal Non-Magic Squares ($n=4, 8, \dots$)

Here the highlight is the discovery (Loly, 2002-2003) of a small new class of purely pandiagonal non-magic, number squares having dimensions of the powers of 2, with the additional property that the binary representation changes by just 1-bit in horizontal and vertical moves (they are counted from 0..15 in the example below):

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

(8)

These squares derive from the Myers-Briggs dichotomous scheme of personality types. Then work with summer undergraduate Marcus Steeds (Loly and Steeds 2003), who devised a number of useful tools in Maple mathematical software, explored how generalizations of these squares are related to the Gray code and square Karnaugh maps of digital logic design, a connection made by some of my third year engineering students a few years ago. I have also applied these ideas to ancient Chinese patterns based on the yin-yang duality (Loly, 2002).

Recently Dan Schindel found that of three million pandiagonal non-magic squares, 48 have the 1-bit property. This agrees with a symmetry argument made recently by Ian Cameron (2003).

10 From Coupled Oscillators to Modern Linear Algebra

Undergraduate physics provides several examples in mechanics and wave motion (from coupled oscillators) where semi-magic **matrices** arise with algebraic or non-integer elements. The investigation of the mechanical problems had a vibrant interplay with mathematics for two centuries from the time of Hughtens and Newton. Hughtens, of course, is well known for his study of the isochrony of pendulum motion. An excellent chronology is found in Brillouin (1946), whose study of waves in periodic systems is a tour-de-force.

A central mathematical theme in physical science and engineering concerns what are known as eigenvalue problems. These involve homogeneous linear equations which only have non-trivial solutions if the **determinant** of the coefficients vanishes, with as many solutions as the number of equations. These issues can be clarified by using a specific example for which the coupled oscillator is ideal. At the same time we can prepare the ground for studying magic square matrices in their own right.

10.1 Homogeneous Simultaneous Equations for the Coupled Oscillator

In the usual description of this one-dimensional problem (Marion and Thornton, 2000) the equations of motion for masses M_1 and M_2 displaced along the x -direction from their equilibrium positions by x_1 and x_2 , a coupling spring of force constant γ , and with each tied to fixed posts at opposite ends by springs of force constant κ are:

$$M\ddot{x}_1 + (\kappa + \gamma)x_1 - \gamma x_2 = 0 \tag{9}$$

$$M\ddot{x}_2 + (\kappa + \gamma)x_2 - \gamma x_1 = 0 \quad (10)$$

These are simplified by taking out a simple time-dependence: $x(t) = B \exp(it\omega)$ for:

$$(\kappa + \gamma - M\omega^2)B_1 - \gamma B_2 = 0 \quad (11)$$

$$-\gamma B_1 + (\kappa + \gamma - M\omega^2)B_2 = 0 \quad (12)$$

Instead of the general approach of setting the determinant of the coefficients of these simultaneous equations to zero, this simple problem may be solved simply by forming ratios of the variables:

$$\frac{B_1}{B_2} = \frac{\gamma}{(\kappa + \gamma) - M\omega^2} = \frac{(\kappa + \gamma) - M\omega^2}{\gamma} \quad (13)$$

Cross multiplication of the second equality results in a quadratic equation in ω^2 , with two solutions, one, $\omega^2 = \frac{\kappa}{M}$, just the frequency of each oscillator without coupling, and the other higher, $\omega^2 = \frac{\kappa + 2\gamma}{M}$. Extended to a chain of two or more different alternating masses and springs, we have the origin of the gaps in the spectrum which are a characteristic feature of solid state physics. We continue this example after introducing some essential matrix operations, which are easy enough to cover in high school.

11 Determinants and Matrices

Cayley initiated matrix theory in 1846, followed by contributions from Peirce, Hamilton, Poincaré, and Sylvester. We highlight the issues of interest with respect to magic squares, the semi-magic matrices and tensors arising in mechanics, and the related interest for pandiagonal non-magic squares with a brief discussion using 2×2 matrices.

11.1 Matrix Multiplication

Consider the usual matrix-vector multiplication:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} aP + bQ \\ cP + dQ \end{bmatrix} \quad (14)$$

Clearly if $P = Q = 1$, this sums the rows:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a + b \\ c + d \end{bmatrix} \quad (15)$$

This $[1, 1]$ vector will be referred to as a diagonal (or 2-agonal) vector, and generalizes to higher orders. However if one takes the original matrix operator

to act from the right onto a row vector on the left, as a "left-hand" problem, then one finds the column sums of the original matrix:

$$\begin{bmatrix} P & Q \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} Pa + Qc & Pb + Qd \end{bmatrix} \quad (16)$$

if $P = Q = 1$, this sums the columns.

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a + c & b + d \end{bmatrix} \quad (17)$$

This may also be achieved by left multiplication of the transposed matrix with a row vector as illustrated next.

11.2 Eigenproblems - Eigenvectors and Eigenvalues

An illustration of the utility of the " n -diagonal" eigenvector $[1, 1, 1, \dots]$ of the n -cube is seen by showing how the rows sum in the Lo-shu magic square :

$$\begin{bmatrix} 3 & 5 & 7 \\ 4 & 9 & 2 \\ 8 & 1 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ 15 \end{bmatrix} = 15 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (18)$$

where we have factored out the eigenvalue, 15, to show the action of the matrix operator in leaving the eigenvector unchanged. The columns have the same eigenvalue as follows from the transposed matrix:

$$\begin{bmatrix} 3 & 4 & 8 \\ 5 & 9 & 1 \\ 7 & 2 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ 15 \end{bmatrix} = 15 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (19)$$

Alternatively we can work with a row eigenvector on the left with the original matrix:

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 3 & 5 & 7 \\ 4 & 9 & 2 \\ 8 & 1 & 6 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 15 & 15 & 15 \end{bmatrix} = 15 \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad (20)$$

We must note that the n -agonal eigenvector is both a left and a right eigenvector, and that this property depends only on the semi-magic property. An immediate application is now afforded by the coupled oscillator.

12 Coupled Oscillator Eigenvectors

When written in modern matrix notation the semi-magic nature of the characteristic matrix: is apparent:

$$\begin{bmatrix} \kappa + \gamma - M\omega^2 & -\gamma \\ -\gamma & \kappa + \gamma - M\omega^2 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = 0 \quad (21)$$

It is immediately clear that a solution is the $[1, 1]$ eigenvector, both as a right, and as a left eigenvector, with the expected eigenvalue for ω^2 . Substituting the other solution into the ratio equation gives $B_2 = -B_1$, or the eigenvector $[1, -1]$.

12.1 Left and right eigenvectors

Of significant interest for our studies of magic squares, this topic is important for teaching linear algebra beyond the introductory course. The use of magic square examples already occurs in such courses, but it can be used even more seriously. The existence of identical left and right eigenvectors implies deeper properties giving rise to the theorem of biorthogonality, and the theorem of Perron (Mattingly, 2001). These theorems develop deep links between the left and right eigenvectors and eigenvalues. As such, magic squares give insight into advanced linear algebra.

Software such as Maple, Mathematica and MATLAB can be profitably employed in such studies. Indeed, MATLAB has initiated some of this already by including a function, *magic(n)*, which returns a magic square from one of three algorithms, one each for odd, even and doubly-even cases. The drawback is that the single squares which result are not representative of the richness of the spectrum of magic squares of a given order, save for $n = 3$.

12.2 Eigenvalues of Magic Squares

In another topic with Marcus Steeds as a significant participant, we have nearly finished a study of the eigenvalues of magic square matrices. For the 880 distinct 4×4 's in the 12 Dudeney groups, we find that members of the first six (singular) groups have three distinct eigenvalue patterns, with a subset of the first three groups having three zero eigenvalues, while the last six (non-singular) groups have two further eigenvalue patterns.

Also if the 1-bit pandiagonal squares discussed earlier are treated as matrices they possess just two non-zero eigenvalues.

13 Conclusion

With the conference theme of "Reforming School Science through the History, Philosophy, and Sociology of Science", we conclude with a few closing thoughts about each of these issues.

Some of the history has already been addressed, but further information may be found in a recent book (Swetz, 2000) by Frank Swetz, a mathematics educator, as well as in Clifford Pickover's recent book (Pickover, 2002). Those

sources also enlarge on the philosophical aspects, which began in China as a cosmology, or organizing scheme.

There are opportunities to enrich teaching in classical physics, and likely in quantum physics as well. Certainly more can be done in the context of teaching linear algebra, which can begin in high school.

As for sociology, I have found wonderful opportunities for students to cooperate in some group work as summer research assistants.

TPT: The Physics Teacher

14 Acknowledgments

David Lavis, Joe Williams, Walter Trump, John Hendricks, and Harvey Heinz have helped with advice on various aspects of this study, which engaged the following undergraduates: Marcus Steeds, Wayne Chan, Adam Rogers, Matt Rempel and Dan Schindel.

This research was funded in 2003 by the Winnipeg Foundation Post Secondary Grants Program, especially for support to Adam Rogers and Matt Rempel.

15 References

References

- [1] Brillouin, L.
- [2] chan : Chan, W. and Loly, P. D. Iterative Compounding of Square Matrices to Generate Large-Order Magic Squares, *Mathematics Today (IMA)*, 38(4), 113-118, August 2002. A colour image can be viewed on my website at: <http://home.cc.umanitoba.ca/~loly/smallmath.jpg>.
- [3] Descombes, René., 2001, *Les Carrés Magiques, Histoire, théorie et technique du carré magique, de l'Antiquité aux recherches actuelles*. Vuibert.
- [4] Heinz, Harvey D. and Hendricks, John R. (2000) *Magic Squares Lexicon: Illustrated*, published by HDH as demand indicates. 15450 92A Avenue, Surrey, BC, V3R 9B1, Canada, hdheinz@istar.ca [ISBN 0-9687985-0-0]. Available from John R. Hendricks, #308, 151 St. Andrews St., Victoria, B.C., V8V 2M9, Canada. For the fourth order squares see: H. Heinz, www.geocities.com/~harveyh/Downloads/MS4_List-Group.zip
- [5] Hendricks, John R. (1992) *The Magic Square Course*, 2nd edition, privately printed.

Many of Hendricks results have been included in Clifford Pickover's *The Zen of Magic Squares, Circles, and Stars - An Exhibition of Surprising Structures across Dimensions*. Princeton University Press (2002) .

- [6] Hruska, F. Magic Squares, Matrices, Planes and Angles. *J. Rec. Math.* vol. 23(3), 183-189, 1991.
- [7] Loly, P. D. A purely pandiagonal 4*4 square and the Myers-Briggs type Table, *J. Rec. Math.*, vol. 31 (1), 29-31, 2002-2003
- [8] Loly, P. D. A Logical Way of Ordering the Trigrams and Hexagrams of the Yijing. *The Oracle: The Journal of Yijing Studies*, 2 (12), 2-13, 2002.
- [9] Loly, P. D., The Invariance of the Moment of Inertia of Magic Squares. *The Mathematical Magazine*. March 2004.
- [10] Loly, P. D., Hruska, F., Williams, J., and Steeds, M. Eigenproperties of Magic Squares. [preprint 2002].
- [11] Marion, J.B. and Thornton, R.K., *Classical*
- [12] Mattingly, R. Bruce. Even Order Regular Magic Squares are Singular. *American Mathematical Monthly*, 107, November 2000, 777.
- [13] Myers, Isabel Briggs with Peter B. Myers. *Gifts Differing: Understanding Personality Type*. CPP Books, Palo Alto, California, 1980, 1993.
- [14] K. Ollerenshaw and D. S. Brée, *Most-perfect pandiagonal magic squares: their construction and enumeration*. Southend-on-Sea, U.K.: The Institute of Mathematics and its Applications (1998).
- [15] Pickover, Clifford. (2002) *The Zen of Magic Squares, Circles, and Stars - An Exhibition of Surprising Structures across Dimensions*. Princeton University Press.
- [16] K. Pinn and C. Wiczerkowski, Number of Magic Squares from Parallel Tempering Monte Carlo. *International Journal of Modern Physics C*, Vol. 9, No. 4, 541-546 (1998).
- [17] Stroepel, Richard
- [18] Swetz, Frank J. (2002) *Legacy of the Luoshu - The 4000 Year Search for the Meaning of the Magic Square of Order Three*. Chicago: Open Court.
- [19] Trump, Walter
- [20] von Franz, M. L., *Number and Time*

Magic squares of squares. Bimagic squares using distinct integers. Normal bimagic squares (using consecutive integers). In a series organized by the University of Picardy, the ESIEE Amiens, the URISP-CNISF, and the ADCS in March-April 2005, several scientific lectures were presented in Amiens to mark the 100th anniversary of the death of Jules Verne. Jules Verne (Nantes 1828 - Amiens 1905), Edouard Lucas (Amiens 1842 - Paris 1891). During this event, I gave a lecture titled "Resort to computing on a problem of Euler and of Lucas": Slides 1-3: Jules Verne and Edouard Lucas were contemporaries, and both men lived for many years in Amiens. Slides 4-13: summary of previously published articles on the "magic squares... Imagine a magic square where each cell is a square tower equal to the height of the cell's numerical value. Craig Knecht refers to this model as a topographical magic square. He has explored this type of magic square for the feature of how much water would be retained in the central part of the square. [1] Peter D. Loly, Franklin Squares: A Chapter in the Scientific Studies of Magical Squares, Complex Systems, 17 (2007) 143-161 [2] Some of his other papers on magic squares are at <http://home.cc.umanitoba.ca/~loly/>. Gaspalou Enumerations. Has a site at <http://www.gaspalou.fr/magic-squares/> with much material on the methods and tools for enumeration of magic squares. Magic squares became very popular in Europe during the middle ages. One of the main characters responsible for this was Cornelius Agrippa (1486-1535), a rather extraordinary German. At university, Agrippa secretly formed a group of students who studied magic and Alchemy. Some of his friends were actually burned alive at the stake because of their dealings with black magic. Another famous German who was also intrigued with magic squares and lived at the same time as Cornelius Agrippa was Albrecht Dürer, a famous artist and mathematician. Dürer included many mathematical and geometrical themes in his artwork, using a compass and a ruler so that his works would be very precise. He also was one of the first artists to study and use proportion in his drawings. Magic squares have fascinated mathematicians for thousands of years. Now Lee Sallows's 'geomagic squares' offer an intriguing new perspective on them. During one acid trip he started doodling with magic squares, a type of mathematical pursuit he had recently read about in a scientific magazine. A magic square is a grid of numbers for which every line, column and diagonal adds up to the same number. Some great mathematicians studied magic squares such as Leonhard Euler in the 18th century, and Edouard Lucas and Arthur Cayley in the 19th but the field has generally been the domain of passionate laypeople. The most notable aficionado was United States founding father Benjamin Franklin, who liked to spend his spare time constructing particularly innovative variations. 3 Semi-Magic and Pandiagonal Magic Squares. For a number of purposes it is important to recognize that there are two specially important variations on the theme of magic squares. Firstly, semi-magic squares, which do not necessarily have the diagonals summing to the row-column line sum, some of which may be obtained simply by moving an edge row and/or column to the opposite side, e.g. 924 573 168. (3). Of significant interest for our studies of magic squares, this topic is important for teaching linear algebra beyond the introductory course. The use of magic square examples already occurs in such courses, but it can be used even more seriously.