TECHNOLOGY IN SECONDARY SCHOOL MATHEMATICS —
THE GRAPHICS CALCULATOR AS PERSONAL MATHEMATICAL ASSISTANT

Barry Kissane
 Murdoch University, Western Australia

ABSTRACT: Although much has been written about technology in mathematics, much less has happened in schools. A major reason for this is the limited availability of computer hardware, not only in lesser-developed countries. Discussion among teachers both within and between countries is hampered by different metaphors for technology. These metaphors are described and their significance explained. It is suggested that regional use of technology is likely to be more effective if emphasis is placed upon graphics calculators rather than on microcomputers. The portability of graphics calculators is a key element. As well as economic and curriculum impediments to change, the central rôles of teachers and the need for effective support is acknowledged.

Mathematics education at the secondary school level in Australia has been surprisingly unaffected by technological developments of the past decade. As for other schools across the Asia-Pacific region, for many Australian schools the reality of information technology in mathematics falls very far behind the more enthusiastic writing on the subject. Even in more affluent countries, computers continue to be relatively scarce (compared with student population numbers), continue to be awkward to access, both physically and in the timetabling sense, and continue to be mainly used for demonstration purposes, where they are used seriously at all. They are rarely integrated into school curricula or associated text materials.

Even in locations where hardware is relatively plentiful, such as in schools using laptop computers, mathematical uses are substantially constrained by available and affordable software, and are much outweighed by word processing. In some cases, portable computers have been purchased in part on the assumption that they will be of value to the mathematics curriculum, but expectations have far exceeded usage, and the limitations of generic (affordable) tools such as spreadsheets have become clear.

Dissuaded by practicalities, typical secondary mathematics teachers in Australia have not embraced microcomputers as everyday devices for teaching, learning or doing mathematics. Rather, they have been relegated to the position of occasional media for demonstration of useful ideas to a whole class, notably concerned with graphing of functions and perhaps with statistical analysis. The situation is no better elsewhere. For example, Ruthven (1993) noted that there was little use made of computers in classrooms in the UK:

Almost invariably, when the machine is used it is as a mathematical tool or as an alternative medium within a thoroughly conventional framework. After ten years of very considerable effort at innovation in the UK, most teachers still lack the training and resources which might enable them to turn new pedagogical forms into an everyday reality. (p. 199)

For some teachers, the struggle to find the right software has been too hard, while for others the fight to access the available school facilities has been too daunting, with economic constraints substantial in each case. In addition, many teachers throughout the region have limited personal experience with technologies for doing mathematics. It has simply been too much trouble for already overworked professionals to change existing curriculum and teaching practices to accommodate microcomputers. It is arguably not yet necessary to do so in view of the actual resource base of most schools, and the external curricular constraints within which teachers work. Regional experience, especially in the less economically advantaged parts of the Asia-Pacific region, has been similarly constrained by resources, so that the actual im-
pact of newer technologies on the daily work of teachers and their students has been minimal. Although part of the slowness of response can be explained by technical and economic reasons, the most important reasons are likely to be human.

**ON BEING SMALL**

Practical and human impediments to adequate consideration of information technologies in mathematics curricula may change substantially with the continuing development of graphics calculators. Unlike other recent developments at the cutting edges of technology, graphics calculators are more likely to be affordable to individual students, available in useable quantities within schools, genuinely personal and portable, and of direct relevance to many parts of secondary school mathematics courses. It is an error of phraseology to describe a modern graphics calculator as anything less than a computer. Rather than regard them as powerful calculators, it is more helpful to regard them as small computers with inbuilt software dedicated to mathematical tasks — as ‘personal mathematical assistants’, the mathematical versions of the PDA’s (personal data assistants) currently fashionable in the information technology industry. Pratt (1992) observed that the abbreviation ‘PC’ for personal computer is a misnomer, since most of the objects referred to were anything but personal, and certainly not able to be conveniently carried on one’s person. The graphics calculator, in Pratt’s view, is a ‘TPC” or ‘truly personal computer’. Although the software in graphics calculators is primitive by today’s computer standards, and the machines are much less flexible than are microcomputers, they are potentially much more available to individual students.

A thesis of this paper is that computing technology will become important to secondary mathematics education through a process of increasing sophistication of graphics calculators, designed to be used by individual students of mathematics, rather than through the opposite process of continuing miniaturisation of multi-purpose computers used for mathematical ends. Hardware accessibility and portability are critical to future development in the schools, homes and minds of pupils, and have so far been grossly underestimated in importance. Seymour Papert described the computer laboratory as the school’s defence mechanism against the invasion of the computer; to date, it has been a very successful form of defence. However, the graphics calculator is small enough to slip through this defence, and indeed a computer laboratory can now fit in a briefcase, with all the resultant advantages of portability. (Bradley, Kemp & Kissane, 1992)

The limited penetration of computers into school mathematics and the hearts and minds of mathematics teachers has affected interpretations of computers in the context of mathematics. The ways in which teachers, students and others think about computing technology depends on the availability of hardware and their associated experiences with hardware and software. These conceptions themselves are important if we are to understand the generally lukewarm reception technology has received in the bulk of mathematics classrooms. It seems likely that communication difficulties arise when different people interpret technology for mathematics education through different eyes. The next section elaborates this idea in some detail.

**METAPHORS FOR TECHNOLOGY**

Metaphors help us to think about new things in the context of older, more familiar things. As a consequence, they can be quite expansive, even liberating. On the contrary, however, they can be stultifying and constraining too, shackling us to the past. Frequently, metaphors are quite subconscious; we don’t know of the metaphors we have until something happens to draw our attention to a limitation brought about by a particular metaphor. All metaphors break down eventually, since there are differences as well as similarities between things.

Experience with technology in mathematics education over several years, and observations of the ways in which people respond to new technologies have suggested to me that there are a number of metaphors in common use that help to make sense of these responses. These are elaborated below. The focus of the elaborations is on graphics calculators, but similar insights can be attained from using the same metaphors to think about other common forms of classroom technology, particularly four-function calculators in the primary school, scientific calculators in the secondary school and microcomputers across the entire spectrum of mathematics education.

The description of each metaphor below begins with remarks of the kind that teachers or their students make, and which seem to offer clues to the underlying metaphoric thinking. To explore briefly the extension of these metaphors, a remark in relation to graphics calculators is given along with one related to
four-function calculators.

**Laboratory**

*Explore these functions and their graphs on your calculator.*

*What happens if you keep dividing by ten?*

The metaphor of a laboratory associates calculators with exploration, experimentation, purposeful play, the discovery of new things. It is suggestive of an environment for learning, a playground, and collaboration within a small group. In a laboratory, we expect to find out for ourselves things we didn’t know before, perhaps even things that nobody has known before. We are in control of what we do, and must observe closely the results of our actions. An overriding aspect is that we are engaged in doing something: we are busy thinking, noticing, responding, discovering and acting, in contrast to being busy watching, listening and attending to others. There are elements of unpredictability in laboratories. It is never quite clear in advance what will happen, and so it becomes more difficult to decide ahead of time exactly what we should do and for exactly how long we should be doing it.

A major reason why calculators can support significant mathematical exploration is that there are major elements of mathematics embedded deep within their construction. For example, the graphics calculator incorporates more than one way of representing elementary functions: using symbols, using graphs on the infinite coordinate plane and, on more recent machines, using tables of values. Students exploring these representations has the means to move freely among them, and to begin to make similar kinds of connections in their minds. In the same way, much of the power of the four-function calculator is a symptom of the representations of the real number line it provides. These representations rely mainly on the decimal number system and involve both positive and negative numbers. On some recent elementary calculators, fractions are included as well. A great deal about both the coordinate plane and the real number system can be learned, or at the very least encountered, by people allowed into such laboratories.

In recent years, the idea of mathematics as a laboratory subject has gradually entered conversations in Australian tertiary institutions, in support of the case for more teaching resources. Graf *et al* (1992) have offered a powerful argument for this idea:

... computers and mathematical software work exclusively in the realm of the phenomena; they can only exhibit phenomena. And they are able to show the phenomena even to students who have not yet mastered the theory. ... using mathematical software, students will get to see and know a lot of mathematical phenomena. The mathematical theory then has to explain these phenomena; thus mathematics shifts in the direction of a science which orders, describes and makes understandable facts that are already known and obvious even without explanation. (p. 65)

Fey (1990, p.64) has also written about the use of technology for exploration, and suggested that “... calculators and computers have created a new balance between theorem-finding and theorem-proving”.

**Teaching aid**

*I’ll show them the graph on the graphics calculator.*

*Watch carefully! What will happen if I press ‘equals’ now?*

The metaphor of teaching aid has many associations: the idea of demonstration, the focus of the whole classroom, the efficient instruction of a large group. Teaching aids have often been regarded as unnecessary extras, and are sometimes only used in special circumstances. This is especially so in less economically advantaged countries than Australia. Student teachers sometimes regard teaching aids as essential, to impress college staff, but they are regarded as more dispensable by teachers who were already credentialed. Generally, teaching aids are in short supply: the teacher has one to use, while the students merely watch. In fact, sometimes educational equipment is used as a teaching aid because it is in short supply, rather than because that is the preferred mode of use. There is an element of troublesomeness about teaching aids. They need to be organised well in advance, so are often easily overlooked for more pressing concerns. The metaphor of teaching aid conjures up images of attentive pupils, of well-organised presentation, of lecturing or showing, of quietness and of teacher control.

Recently, overhead projector versions of calculators and microcomputers have become technically sound and (at least in the case of four-function calculators) relatively inexpensive. These are clearly designed to be used as teaching aids, with the main user being the teacher. While advantageous for teaching people how to use the equivalent calculators,
presumably a short-term goal in most cases, it is less obvious that they have significant merit for teaching and learning mathematics itself.

**Tool**

*Use your calculator to solve this system of linear equations.*

*I worked it out on my calculator, Sir.*

The metaphoric associations of tool are particularly strong in mathematics education, but it is important to pay attention to the user of the tool. (See Kissane (1990) for an elaboration of this point in the context of computer as a tool.) We use tools to save our own labour or to do things that we could not readily do otherwise. The kitchen and the workshop are associated with tools, good examples being the whisk and the screwdriver. Tools often amplify our capacities, rather than alter them, but as Dreyfus (1994, p.210) notes, “... computer tools have the potential to contribute to the learning process not only as amplifiers but also, and more importantly, as reorganizers”. In Australia, the metaphor has several colloquial expressions commonly associated with it, such as ‘using the right tool for the job’ and the concept of ‘learning the tools of the trade’. Tools are active things: we do things with tools more often than we use them to think about or to speculate with, although the idea of cognitive tools is powerful and provocative, too (Dörfler, 1993; Dreyfus, 1994). The metaphoric association with a ‘toolbox’ suggests that people often have a variety of tools at their disposal, and it is important to select the right tool as well as to learn how to use it well. Tools are often personal things, especially to skilled practitioners: the cook prefers his own knife and the carpenter is more comfortable with her own saw than with someone else’s.

In the case of graphics calculators, these sorts of metaphoric links are quite suggestive. Some of the new tools are powerful and labour-saving, such as commands that invert a matrix or draw a line of best fit through a scatter plot. To use such tools effectively requires that students make sound decisions about when to do so, as well as learning the particular mechanical skills involved. Such decisions cannot be made for students only by their teachers or their textbook authors, or it will be unlikely that they will develop appropriate decision-making skills for themselves. We may become interested in how well students can use the calculator as a tool, and so may be encouraged to place them in contexts where it is important to do so. We may discourage students from using the wrong tool for a job, such as inverting a 3x3 matrix by hand or using a knife to insert a screw into a piece of wood, when better tools are available. Indeed, we may focus on the intelligent use of tools such as graphics calculators as an outcome of mathematics education in itself, rather than merely a means of achieving other outcomes.

**Curriculum influence**

*When should we teach curve sketching? Should we teach it at all?*

*But, Miss, you can take 8 from 5 – you get minus 3.*

Technology is the most powerful force for curriculum influence today. Calculators change our view of what is a mere computation. For some graphics calculators, a few key steps will multiply matrices, evaluate integrals, solve equations or find points of intersection of two graphs. Today’s calculators for young children deal with negative numbers, fractions and percentages as smoothly as earlier models dealt with decimals. The perspective encouraged by this metaphor has been vividly described:

Just as the introduction of calculators upset the comfortable pattern of primary school arithmetic, so the spread of computers will upset the traditions of secondary and tertiary mathematics. This year long division is passe; next year integration will be under attack. (Steen, 1992, pp. 35-6)

Computers and calculators have changed the world of mathematics profoundly. They have affected not only what mathematics is important but also how mathematics is done. It is now possible to execute almost all of the mathematical techniques taught from kindergarten through the first 2 years of college on hand-held calculators. This fact alone must have significant effects on the mathematics curriculum. ... the changes in mathematics brought about by computers and calculators are so profound as to require readjustment in the balance and approach to virtually every topic in school mathematics. (Romberg, 1992, p.772)

As well as casting doubt on what belongs in the curriculum, graphics calculators influence opinion on the emphasis to be placed on mathematical concepts, skills and strategies and the optimal sequences in which they
ought be treated in the curriculum. To illustrate these kinds of influence, the recent lower secondary school algebra text by Lowe et al. (1994) breaks with long-standing curriculum traditions by paying more attention to iterative solutions of quadratic equations than to exact solutions, and dealing with finding maximum and minimum values of functions long before students encounter differential calculus. Several discussions of this kind of influence in various areas of mathematics are contained in the middle section of Andrews & Kissane (1994).

There are substantial threats to the existing curriculum associated with machines that today invert matrices, solve non-linear equations, give exact solutions to linear systems, perform arithmetic with complex numbers, evaluate definite integrals and graph functions, and that by tomorrow will have acquired further procedural competence well in excess of what most high school graduates of yesterday actually attained. For a spirited discussion of these sorts of matters in the context of computers and graphics calculators with symbolic manipulation capabilities, see Wilf (1982) and Nievergelt (1987). It is hardly surprising that many prefer to ignore such potential curriculum influences altogether rather than try to deal with them, since this is the easier course of action. While hardware and software continue to be beyond the everyday reach of many students, teachers and schools, curricula are unlikely to change to accommodate technological developments. Further, serious impetus to professional involvement with information technology in mathematics education will not be given until curricula are more responsive to technological change. The significance of the graphics calculator is that it may accelerate these mutually interactive processes.

Some ways in which mathematics curricula and teaching methods are affected by the presence of graphics calculators include changes to the content of curricula: re-sequencing of concepts, removal of outdated material, inclusion of new material and early introduction of previously inaccessible ideas (such as relative extrema of functions). Other possible modifications include the use of graphics calculators in assessment and extended investigation work, with associated pedagogical implications. It would seem important to reconsider our goals in the light of technology. (Bishop, 1993) Because of significant differences in accessibility of computers and graphics calculators, curriculum development in this area is likely to have regional, not just local, value.

The influence of technology is not always benign, as Burrill (1992) observes:

It has been claimed that factoring is a major topic in first-year algebra only because it can be taught. Are ideas now being taught because they can be taught using calculators? ... Will we force students to solve systems of linear equations in two unknowns by using the calculator and row-and-column manipulations, even when the system might be far easier to solve with pencil and paper? Or will we teach them to use the inverse matrix without talking about the structure that validates the process? Are we about to trade one set of magic rules for another? (p. 22)

The metaphor of curriculum influence is of special importance to the curriculum developer and textbook author, rôles which to some extent are usually undertaken by all mathematics teachers except those who, whether by choice or by coercion, rigidly adhere to either a particular mandated curriculum or the details of a particular text. The graphics calculator can be seen as a destabilising and hence unwelcome influence by a profession characterised by conservatism and suspicion of change, or as a powerful agent provocateur by those keen to loosen some of the shackles of the past.

Cheating device

Calculators are all very well, but I want them to invert the matrix by hand first.

If I let them use a calculator, they’ll never learn their tables.

Another metaphor for the calculator is as an agent for subversion or impropriety. There is a proper way to do things, a code of behaviour, and so the use of a calculator is seen as akin to cheating. For example, the recent sale in Australia of (scientific) calculators with a facility to solve systems of linear equations raised the spectre of calculator cheating in external examinations, since some students had the new calculators while others didn’t. The observation that calculators must sometimes be used furtively, in preference to doing mathematics the ‘right’ way, serves to underscore the significance of the ‘right’ way in mathematics education. From the beginning of primary school to at least the early undergraduate years, much store is placed on orthodoxy in mathematics, and teachers assume positions of considerable power in defining the orthodoxy. The greater emphasis on ‘proper’ procedures in mathematics than in
many other areas of the curriculum reminds us that for many students mathematics is interpreted as a large body of conventional procedures, rather than a rich and interconnected web of ideas.

Some of the concern for the right way of doing things in mathematics has benign origins, of course. Some mathematical procedures are more efficient, less error-prone or more likely to be insightful than are others. Some of the concern is related to diagnosis and assessment, and particularly partial assessment. Teachers prefer students to record their mathematical work in some detail, so that they can locate any errors in thinking, misconceptions or computational slips, and so award part marks. It is all too easy for the medium to become the message, however, so that the conventional recording and the right way of doing things become the ends in themselves rather than the means to an end, and mathematics can be uncomfortably close to an essentially imitative activity.

The graphics calculator as a cheating device is a significant metaphor because so many of the routines of mathematics can be dealt with by a few steps on a modern calculator, a significant change from its ancestor, the scientific calculator. As well as the number of routines, their nature is important, since parts of the core of the standard algebra-calculus sequence are at stake, rather than mere arithmetic computation. The metaphor is likely to be invoked, implicitly at least if not explicitly, because the graphics calculator allows for new ways of dealing with the solution of equations, sequences and series, graphing, optimisation, matrix arithmetic, integration, differentiation and data analysis. At the same time, the graphics calculator fuels concerns that factorisation, long a sacred cow of elementary algebra, is actually of limited practical significance and even demands a careful look at the balance of discrete and continuous mathematics.

Status symbol

*This calculator has over 430 functions, so it must be better.*

*Your calculator doesn’t even do fractions!*

One of the features of technologies these days is that they seem to change quickly. Last year’s model seems ancient, and next year’s model enticing. We are mindful of the push to have the latest, the newest, the calculator that does something that no other calculator does, or does it better, or does it faster, or does it easier, … We don’t want to get left behind. This metaphor is especially powerful for computers, in well-known ways easily seen in modern advertising. Rather than reduce prices, manufacturers prefer to increase the features offered for the same price, and take advantage of the apparently human drive (at least in industrialised countries like Australia) to acquire the latest and hence the best as soon as possible. Thankfully we have not reverted to the early days of scientific calculators, which came with belt loops, so that they could be proudly displayed in public, mainly by engineers. Although there would appear to be not too much harm done in technophiles eagerly seizing upon new models, there is a risk that this metaphor for technology can mistake the means for the ends. For many students, regular access to and appropriate experience with a reasonably recent model is adequate; access to the very latest models is not essential. There are also clear risks of substantial inequities arising within a school or a school system, unless careful attention to assumptions about access to technology is paid. In this regard, it is interesting to note that for the new versions of the College Board’s Advanced Placement Calculus examinations in the USA, for which graphics calculators are assumed to be available to all students, calculator programs are provided to ensure that students with less powerful calculators are not disadvantaged, and explicit attention is paid to assumed calculator capabilities.

**THE SIGNIFICANCE OF THE TEACHER**

Effective regional collaboration on the use of computing technologies in mathematics has been severely constrained by different interpretations of their significance, reflected in metaphors like those suggested above. There is not space in this paper to elaborate the implications of this observation. However, it needs to be acknowledged that the role of the classroom teacher is central, but easily overlooked in importance. Ruthven (1993, pp. 190-1) has described the “marginal impact of machines on teaching” and “the dissonance between the neoprogressive proscriptions of many of the proponents of educational change, and the current realities of classroom practice”. Cohen (1987) has discussed at length the impediments to what he calls ‘adventurous teaching’, arguing that schools have an ancient instructional heritage with some resilient ideas, of knowledge is purely objective, of teaching and telling as synonyms, and of learning as a passive process of accumulation. Jackson (1987) has de-
scribed two outlooks on teaching, which he calls ‘mimetic’ and ‘transformative’, traced their roots to Socrates and the Sophists, and cautions us not to expect change quickly. Kaput (1992, p.517) has noted that, “Serious use of computers of any kind, let alone the level of computing to be available in the 1990s and beyond, is simply not part of the culture of schools and schooling.” Cornu (1992, p.27) accepts the difficulties of teacher change, in terms of “… a sacrifice of traditional security, at a time when teachers will still be fighting hard to gain new skills and acquire confidence in them. It would be foolish to underestimate the challenge this presents”. There is not space left in this paper to draw out the pedagogical implications of graphics calculators, but it is clear that, from a range of professional perspectives, these and other authors would suggest that we do so carefully.

In other words, secondary mathematics teachers are unlikely to embrace and successfully integrate graphics calculators (or indeed any other forms of technology) into their teaching without considerable help. Some of the help needs to be in the form of curricula designed to support such a movement. Secondary school examples of these have begun to appear recently in industrialised countries: the Nuffield Advanced Mathematics Project (Nuffield Foundation, 1994), The University of Chicago School Mathematics Project (Rubinstein et al, 1992), and Access to Algebra (Lowe et al, 1994). Such work will not suffice however; teachers will need the support provided by personal experience with technology, and the opportunity to work with colleagues to see that there are many helpful metaphors for technology, and that there are grave risks associated with a preoccupation with only one of them. Policy makers ignore these and other real needs of teachers at their peril.

CONCLUSION

Successful incorporation of technology into the secondary school mathematics classroom has so far been elusive. The graphics calculator offers some prospect, at least in the medium term, for a more realistic consideration of the possibilities for curriculum development than we have seen to date. A significant part of the appeal relates to the physical size and hence portability and accessibility of the machine. It seems likely that the many metaphors we use for technology in mathematics teaching can become part of our conversations, if the graphics calculator becomes a regular part of the mathematical lives of teachers and their pupils. The resulting experience may be enough to assist many teachers to overcome many of the real impediments to sound use of modern technologies that support teaching, learning and doing mathematics.

REFERENCES


* This paper was presented to and published in the ICMI 95 Conference Proceedings:

Alice F. Artzt is Professor, Secondary Mathematics Education, Queens College of the City University of New York. Tell the Publisher! I'd like to read this book on Kindle. After checking some other books, I decided to buy The Mathematics that Every Secondary School Math Teacher Needs to Know in its print edition because it is full of good problems that prompt mathematics teachers to think about mathematics deeply and to think about how to communicate clearly to learners. Mathematics teaching would be better in general if more teachers worked through the problems in this book. Read more. One person found this helpful. Approved Calculators for Specified VCE Mathematics Examinations 2013. Use of Technology in Secondary Mathematics. Final Report for the International Baccalaureate. Paul Drijvers John Monaghan. Explore types of technology used in mathematics curricula for students aged 16-19 years old in different educational systems (within a country and internationally); Examine approaches and strategies to technology integration in curriculum design, pedagogy and assessment; Indicate issues involved in the use of technology in mathematics courses; Identify factors increasing the effectiveness of technology implementation in classroom practice; Analyse the. Because imagine not learning math at the secondary levelâ€¦ Weâ€™d have no technological advancement. I have a doctorate from a prestigious university. By the time I got to my first statistics course there, calculators and computers did most of the work that I find so difficult. What I needed to know was the tests to program the computer to produce the data and then how to interpret the data, of course. I did not need to do algebra, geometry, calculus, trig, thankfully. Calculating technology in mathematics has evolved from four-function calculators to scientific calculators to graphing calculators and now to computers with computer algebra system software. The use of CAS in education is still relatively rare but the growing body of research and the interest suggests that its extended use is imminent. They can be used to change the emphasis of learning and teaching of mathematical concepts away from techniques and routine symbolic manipulation towards higher-level cognitive skills that focus on concepts and problem solving. Two of the key indicators of