ALEXANDROV GEOMETRY

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1 Overview of the Project

The purpose of our research stay was to work on our book “Alexandrov Geometry”. Our book is supposed to be a comprehensive text and reference work on the fields of curvature bounded below and curvature bounded above. Although these two fields developed quite independently, they have many similar guiding intuitions and technical tools. Our approach is novel in its attention to the interrelatedness of the two fields, and its emphasis on the way each illuminates the other.

In addition to all the basic material in both fields, the book includes all the important advanced material on spaces of curvature bounded below. This material is unavailable in any book, and not all of it is in the literature. For spaces of curvature bounded above, we are emphasizing topics and proofs inspired by considering the two contexts simultaneously.

2 Progress Made

At present, our draft is about 250 pages. The final version will be at least twice that. Most of the current draft material is in the attached version of the book.

We were working on the book for more than half a year without face-to-face contact. (Before that we met for a week in October 2008 and S. Alexander and A. Petrunin met in July 2009). During this time, a number of issues accumulated. We were able to get through the complete list of them and make very substantial progress in just one week at Banff.

Many of our discussions during the research stay concerned proofs and advanced topics to be included. Some proofs became more transparent, and new and surprising dualities between the two bodies of material came to light. These findings improve the book's coherence and elegance. More specifically, we mostly worked on Chapter 6 (Definitions of curvature bounded from below) and Chapter 7 (Definitions of curvature bounded above). Chapter 6 is now mostly complete and we feel it's sufficiently ready to be made available to public. Therefore, after coming back from Banff we posted this chapter on the web. It can be accessed here http://www.math.toronto.edu/vtk/the-defs-CBB.pdf , here http://www.math.uiuc.edu/sba/the-defs-CBB.pdf or here http://www.math.psu.edu/petrunin/papers/alexandrov-geometry/

Chapter 7 is now also mostly complete and hopefully we'll soon be able to make it available to public as well.

The visit to BIRS had other unexpected benefits:

- Consistency: The fields of curvature bounded below and curvature bounded above can favor different formulations, and a balance must be negotiated.

- Notation: Our approach requires new notations aimed at transparency and economy. Finally settling on optimal notation to mediate between the reader and the material is challenging.
- Technical: Our figure-preparation and file-sharing arrangements were improved.
- Lastly: We had many interesting conversations with other BIRS visitors at mealtimes and while they were not usually directly related to our main project they were nevertheless quite stimulating.

  Overall we had an extremely productive visit which was absolutely perfect for the kind of project we are involved in.

[KM] works with flows in a more general setting and does not mention CAT(K) geometry and gradient curves. There are flows to which our method applies that do not satisfy the conditions in [KM].Â This chapter is not where to start reading this book, which rather is Chapter 2. The material in the present chapter is meant to be used as a reference for some background material and ideas from linear algebra, which are essential to this book, in particular to the first part of it on algebra and geometry consisting of Chapters 2 through 5. In geometry, Alexandrov spaces with curvature â‰¥ k form a generalization of Riemannian manifolds with sectional curvature â‰¥ k, where k is some real number. By definition, these spaces are locally compact complete length spaces where the lower curvature bound is defined via comparison of geodesic triangles in the space to geodesic triangles in standard constant-curvature Riemannian surfaces. John Harvey, â€œEquivariant Alexandrov Geometry and Orbifold Finitenessâ€, J Geom Anal, 2015. Nina Lebedeva, â€œAlexandrov spaces with maximal number of extremal pointsâ€, Geom. Topol, 19:3 (2015), 1493.Â Shioya T., Metric Measure Geometry, Gromov's Theory of Convergence and Concentration of Metrics and Measures, Irma Lectures in Mathematics and Theoretical Physics, 25, Eur. Math. Soc., 2016. Alexandrov geometry can use â€œback to Euclidâ€ as a slogan. Alexandrov spaces are dened via axioms similar to those given by Euclid, but certain equalities are changed to inequalities. Depending on the sign of the inequalities we get Alexandrov spaces with curvature bounded above or curvature bounded below. The denitions of the two classes. Contents.